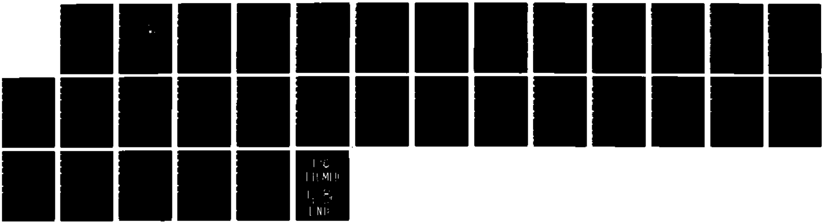
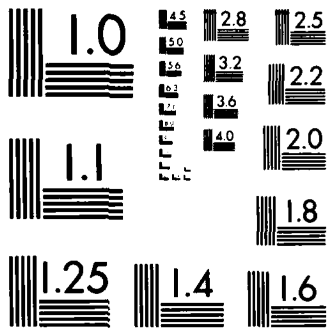
AD-A165 186 ESTIMATING RELATIONSHIPS IN SIMULATION MODELS USING REGRESSION: AN APPLIC. (U) NAVAL POSTGRADUATE SCHOOL MONTEREY CA K J EUSKE ET AL. SEP 85 NPS-54-85-010 1/1

UNCLASSIFIED MIPR-DWAM-50023 F/G 12/1 NL





MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

(2)

NPS-54-85-010

NAVAL POSTGRADUATE SCHOOL

Monterey, California

ESTIMATING RELATIONSHIPS IN SIMULATION MODELS USING REGRESSION: AN APPLICATION TO MILITARY RETIREMENT COSTING

by

K.J. Euske, G.W. Thomas, and LCDR D.F. Smith

Prepared for: The Office of the Assistant Secretary of Defense (MRA&L), Washington DC 20350

Naval Postgraduate School

Monterey, California

Rear Admiral R.H. Shumaker
Superintendent

D.A. Schrady
Provost

The work herein was supported in part by the Office of the Assistant Secretary of Defense (MRA&L).

This report was prepared by:

George W. Thomas
Associate Professor, Department of Administrative Sciences

Kenneth J. Euske
Associate Professor,
Department of
Administrative Sciences

LCDR Donald F. Smith Jr., SC, USN
Department of Administrative Sciences

Reviewed by:

Willis R. Greer, Chairman
Department of Administrative Sciences

Released by:

Kneale T. Marshall
Dean of Information and Policy Sciences

unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NPS 54-85-010	2. GOVT ACCESSION NO. AD-A165186	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Estimating Relationships in Simulation Models Using Regression: An Application to Military Retirement Costing.		5. TYPE OF REPORT & PERIOD COVERED 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) George Thomas Kenneth Euske LCDR Donald Smith		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, CA 93943		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS MIPR # DWAM50023
11. CONTROLLING OFFICE NAME AND ADDRESS Office of the Assistant Secretary of Defense (Manpower, Installations, and Logistics)		12. REPORT DATE September 1985 13. NUMBER OF PAGES 25
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) (see over)		

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

S/N 0102-LF-014-6601

unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

This article demonstrates a procedure for using regression analysis to develop estimating equations that simply and quickly predict the effects on system performance of changes in parametric values of complex interrelationships imbedded in a full simulation model. Though an analytic solution to the relationships may be possible, the time and other resources necessary to generate such a solution may exceed those available. The resulting estimating equations facilitate understanding of the system being simulated, enable users to more easily conduct sensitivity analysis and answer what-if questions, and assist the "selling" of the simulation results to potential users. The procedure includes criteria for selection of: variables to be analyzed, the sensitivity range, the value increments, and the functional form. The example utilized is a simulation model developed for estimating future military retirement costs.

ESTIMATING RELATIONSHIPS IN SIMULATION MODELS USING REGRESSION: AN APPLICATION TO MILITARY RETIREMENT COSTING

by

K.J. Euske
Associate Professor of Accounting
Naval Postgraduate School
Monterey, California
(408) 646-2860

G.W. Thomas
Associate Professor of Economics
Naval Postgraduate School
Monterey, California
(408) 646-2741

and

LCDR Donald F. Smith Jr., SC, USN

August 1985

Funding for this research was provided by the Office of the Assistant Secretary of Defense (Manpower, Installations, and Logistics). The views expressed in this paper do not necessarily represent those of the Department of Defense.

ESTIMATING RELATIONSHIPS IN SIMULATION MODELS
USING REGRESSION : AN APPLICATION TO MILITARY RETIREMENT COSTING

ABSTRACT

This article demonstrates a procedure for using regression analysis to develop estimating equations that simply and quickly predict the effects on system performance of changes in parametric values of complex interrelation-ships imbedded in a full mathematical simulation model. Though an analytic solution to the relationships may be possible, the time and other resources necessary to generate such a solution may exceed those available. The resulting estimating equations facilitate understanding of the system being simulated, enable users to more easily conduct sensitivity analysis and answer what-if questions, and assist the "selling" of the simulation results to potential users. The procedure includes criteria for selection of: variables to be analyzed, the sensitivity range, the value increments, and the functional form. The example utilized is a simulation model developed for estimating future military retirement costs.

ESTIMATING RELATIONSHIPS IN SIMULATION MODELS
USING REGRESSION: AN APPLICATION TO MILITARY RETIREMENT COSTING

One of the main strengths of a mathematical model that represents real systems is its ability to provide insight into the cause-and-effect relationships within the system. The model abstracts the essence of the problem and reveals underlying structure of the system . Often, however, it is not possible to construct a mathematical model that is both a reasonable idealization of the problem and amenable to solution. Some real-world situations are very difficult to represent in concise models because of complexity and stochastic relationships. In these instances, simulation often provides the only practical approach to a problem.

It is often necessary to obtain generalizations of simulation results to facilitate understanding of the system being simulated, the "selling" of the simulation results to potential users, and to enable users to more easily conduct sensitivity analysis (Kleijnen, 1977). A simulation describes overall behavior of a system in terms of interrelationships of individual parameters of the system. The specific relationship between a parameter of the system and the overall performance of the system is embedded in the simulation model. And, unfortunately, the complex relationships of the system are often not tractable to straightforward analytical simplification resulting in the resources required to generate the analytic solution exceeding the potential benefits. In these instances, it may be possible to use regression analysis to develop simplified estimating equations that impound the cause-and-effect relationships reflected in the simulation model and, hence, permit the prediction of the system's behavior.

The purpose of this paper is to present guidelines for using a regression procedure to estimate equations that simply and quickly predict the result of changes in parameter values of complex interrelationships embedded in a mathematical simulation model. The example used is a simulation model developed for estimating future military retirement costs.[1] The ability to evaluate the impact on retirement costs of changes in the underlying assumptions and parameters is important because manpower policy makers are expected to include retirement costs in their manpower decisions and therefore must estimate the responsiveness of retirement costs to alternative decisions.

The procedure utilized in this effort is a simplified development of regression metamodels for generalizing simulation results (Kleijnen, 1977; Friedman and Friedman, 1984). In brief, the process used to develop the estimating equations from the simulation model was:

1. select independent variables for analysis
2. identify the functional form of the relationship of the independent variable to system performance
3. select range and step values for variables of interest
4. select baseline values for the full simulation model
5. run the full simulation model for each of the variables of interest of the range selected
6. use the output of the previous steps to develop the regression equations.

The following sections present the example simulation model and then discuss criteria for selection of: variables to be analyzed, the functional forms to be tested in the regression phase, the range over which the analysis will be conducted, the step values to use for each parametric analysis, the selection of the baseline values for the simulation, and the regression equations resulting from the analysis. Additionally, the estimation of joint effects is dis-cussed.

SIMULATION MODEL

The costs currently being incurred for future military retirees have been accounted for on a pay-as-you-go basis which has tended to understate the costs of current manpower decisions (Aeila, 1980; CBO, 1978). In order to capture the full cost of manpower decisions, actuarial estimates of future retirement costs have been developed (U.S. Department of Defense, 1983). Waterman's review of accounting procedures used in the private and military sector to account for pension costs concluded that the individual entry age normal method is the most appropriate for modelling retirement cost changes for the active military (Waterman, 1983). Waterman's mathematical model along with a set of baseline economic and actuarial assumptions were used to develop a computer simulation model to estimate future retirement costs (Smith, 1983).[2]

The entry age normal technique levies a constant amount for each year's employment. Differences between actuarial assumptions and actual outcomes can cause gains and losses to the fund and therefore impact the flat rate assessment. This flat rate assessment, called the normal cost, is adjusted for the value of these minor gains and losses by amortizing them over the remaining working life of the participants (Dreher, 1967). Individual entry age normal calculations use the variables listed in Appendix A. In addition to the general form age entry normal variables, the military-retirement-specific variables shown in Appendix A are required in the calculation of military retirement.

VARIABLE SELECTION

Once the simulation model was operating, the regression equations could then be developed. The first step was to select the variables for analysis. The variables for which estimating equations were desired consist of two types. The first type are the discretionary management policy variables. The second type of variables are those that reflect important environmental factors that are likely to change. Even though the second type of variable is uncontrollable from the viewpoint of management, its impact on overall system performance must be predicted in order to conduct planning properly for future periods.

The following type-one variables were chosen for parametric analysis: length of service at retirement, probability of an entrant reaching retirement, length of service required, maximum percentage of base pay allowed for retirement, percent of base pay earned for retirement, and rate of salary increase. None of these variables are completely controllable by management, yet each of them can be altered by management policies affecting hiring rates, promotion/retention rates, retirement eligibility, and compensation. The type-two variables include, the annual discount rate and life expectancy at retirement.

The "controllability" of a variable is dependent upon the managerial level which can influence it. For this analysis controllability is defined as having administrative or legal control. For instance, the annual discount rate is relatively uncontrollable at all levels throughout the government because its value is determined by the external market forces which affect not only

the cost of government borrowing but also the cost of private debt. The rate of salary increase is controlled by Congress and is tied to both the projected inflation rate and Congressional perception of military retention. The length of service at retirement and length of service required to retire can be controlled by stretching time-in-service requirements for advancements and by adjusting the current acceptable time-in-service for retirement (currently 20 years), up to the legal maximum of 30 years with no Congressional action. Life expectancy at retirement is a function of multiple environmental effects. The entrant retirement probability is controllable by the military services by adjusting advancement opportunities and by reduction-in-force actions. Both the percent of base pay at retirement and the maximum allowed percent of base pay, are legislated by the Congress and, therefore, though controllable at a higher level, are uncontrollable by the Department of Defense.

SELECTION OF FUNCTION FORM

Given the set of variables chosen for parametric analysis, the function form could be selected. Two distinct procedures were used in the selection of functional form. The first was an analysis of the basic relationships in the model. The variety of functional forms used in these relationships signaled a set of forms to be tested for the regression analysis. For instance, the basic functional relationships in the entry age normal equation include linear, polynominal, and log-linear forms. The second procedure used for selecting functional forms was exploratory data analysis (Tukey, 1977).

RANGE AND STEP VALUE SELECTION

Variable range selection is mainly based on precedent. When simulating a system that has sufficient history, recent data can be used to bound the likely values for changes in parameters. The variables selected for analysis of military retirement costs fall into this category. An additional concern is to select a wide enough range to include anticipated policy changes. In the example, this was particularly important for the minimum length of service necessary for retirement eligibility. If a system without a history is simulated, the selection of a variable range is much more subjective and would probably necessitate an expert judgment procedure to arrive at acceptable ranges. In any event, the procedure should use bounds that are consistent with worst-case, best-case scenarios.

For example, the range for the discount rate (the government borrowing rate) was 5 percent to 15 percent. This range encompassed the historical performance of the 1960's (5 percent) and the possible interest rates of a high inflationary period like the early 1980's (15 percent). Value increments were chosen on the basis of generating ten or more data points for the ensuing regression analysis. Sampling from the permissable values used a normal distribution centered on the most likely value. The bounds and step values chosen for each of the variables selected for the analysis are discussed in later sections.

Once the previous steps were completed, the simulation model was used to develop the output for each of the variables at the selected values. Regression analysis was then used to estimate analytical relationships between total retirement cost and each of our selected variables. A criterion for the existence of a usable functional relationship was a coefficient of determination (R^2) of .90 or higher. The output was used not only to develop

the regression equation but also to conduct a sensitivity analysis of the relationships.

INITIAL VALUE SELECTION

The initial input values selected to establish the baseline for the sensitivity analysis are those used by the Defense Actuary in the Fiscal Year 82 Valuation of Military Pay, (U.S. Department of Defense, 1983) with the single exception of the recommended discount rate. The Department of Defense Actuary recommends a rate of 6 percent which is approximately the average yield on long term U.S. securities for the period 1960 through 1978. After review of recent trends in long term U.S. government securities, a discount rate of 9 percent was selected because the average interest rate has been 9.09 percent for 20 year U.S. government treasury securities for the period 1973 to 1983 (U.S. Department of Defense, Summer 1983).[3]

Table 1 lists the input variable values, their abbreviations, their controllability, and the resulting total retirement cost (TRC) (in millions of dollars), when calculated for the Selected Baseline configuration. The input values for those variables ending in "%" are the percentages used in computation (e.g., a SAL% of 5.5 percent means a salary increase rate of 5.5 percent was used in computation). Input values for variables ending in "D" are the incremental difference between the Department of Defense Actuary's specific estimates and the amounts used in computation (e.g., an LEXPD of +1 means the actuarially computed life expectancies at retirement were all extended by one year). The incremental difference values for those

actuarially computed variables are listed since each paygrade's individual actuarial data (e.g., the life expectancy of a retiring 44 year old senior enlisted is 30.24 years compared with 33.54 years for a 44 year old senior officer) was used to provide greater accuracy. The length of service required to retire (MLOS), is shown at its absolute input value and is neither a percentage nor a change.

TABLE 1

BASELINE VALUES

Description	Abbreviation	Controllable Fed.Govt.	Controllable DOD	Baseline Value
Annual Discount Rate	DIS%	No	No	9%
Estimated Rate of Salary Increase	SAL%	Yes	No	5.5%
Percent of Base Pay at Retirement	PAY%	Yes	No	2.5%
Maximum Allowed Percent of Base Pay	MAX%	Yes	No	75%
Length of Service Required to Retire	MLOS	Yes	Yes	20 yrs
Length of Service at Retirement	LOSD	Yes	Yes	0
Life Expectancy at Retirement	LEXPD	No	No	0
Entrant Retirement Probability	ERPD	Yes	Yes	0

Total Retirement Cost $1,210 M

Table 1 may be interpreted as follows for the Selected Baseline: If one assumes a discount rate of 9 percent, salary growth of 5.5 percent, a minimum of 20 years of active service for retirement with retirement pay equal to 2.5 percent of the retirement basis per year served and a maximum rate of retirement pay not exceeding 75 percent of the basis, and retention and longevity statistics as computed by the Department of Defense Actuary, the cost which should be accrued in 1983 to cover year groups 1953 through 1982 for regular Navy personnel[4] (excluding disability and survivor benefits) is $1,210 million.

RESULTS

Table 2 presents the derived estimating equations. This table displays the variable, its total annual retirement estimating equation, the associated coefficient of determination (R^2) and the increments (Inc) used to generate the output from the simulation. Each equation is discussed below.

TABLE 2

SELECTED BASELINE ESTIMATING EQUATIONS

Variable	Total Annual Retirement Cost Estimation Equation (In Millions)	R^2	Inc
DIS%	Antilog ($ 9.15 - ($.226 x DIS%))	.999	1%
SAL%	$501.3 + ($133.3 x SAL%)	.991	0.5%
PAY%	$ .406 + ($484.6 x PAY%)	.999	0.1%
MAX%	-$895 - ($60.2 x MAX%) - ($0.429 x MAX%^{2})	.989	2.5%
MLOS	$818.9 + ($ 19.6 x MLOS)	.940	1%
LOSD	$1,219 + ($ 19.7 x LOSD)	.952	0.6%
LEXPD	$1,214 + ($ 5.54 x LEXPD)	.994	0.5%
ERPD	$1,212 + ($ 71.7 x ERPD)	.999	0.006%

Discount Rate

The discount rate (DIS%) was examined over a range of 5 percent through 15 percent in 1 percent increments. The function which gave the highest coefficient of determination (R^2 = 99.9) was a logarithmic function in TRC = $9.15 -($0.226 * DIS%).

Salary

The rate of salary increase (SAL%) was examined over a range of average annual increase from 2.5 percent through 7.5 percent in .5 percent increments. An R^2 of .991 was calculated from the linear function TRC = \$501.3 + (\$133.3 x SAL%).

Percent of Base Pay at Retirement

The percent of base pay (PAY%), is currently set at 2.5 percent per year of active duty completed, with a minimum of 20 years duty required for retirement. The total retirement amount cannot legally exceed 75 percent of the final active duty base pay. The percent of base pay (PAY%) was examined over a range of 2.0 percent through 3.0 percent in .1 percent increments. The baseline configuration with a TRC of \$1,210M is the PAY% value of 2.5 percent.

There is a clear relationship between reductions in percent of base pay and resulting annual retirement costs: a given percentage reduction in the first results in an equal percentage reduction in the latter. The linear function TRC = \$ .406 + (\$484.6 * PAY%) provided an R^2 of .999.

Maximum Allowed Percent of Base Pay

The maximum percent of base pay (MAX%) is currently set by law at 75 percent of the final active base pay and is the upper bound. The maximum percent of base pay, was examined over a range of 50 percent to 75 percent in 2.5 percent increments. The baseline configuration with a TRC of \$1,210M is the MAX% value of 75 percent. The results indicate a response pattern of an initially strong positive relationship tapering to almost no effect at the higher end of the MAX% range. The effect is caused by the relatively few service members who remain for a full 30 year retirement and thereby encounter

the 75 percent restriction. An R^2 of .989 resulted from the parabolic function TRC = -\$895 + (\$60.2 x MAX%) -(\$0.429 x MAX%^{2}).

Minimum Length of Service To Retire

The current minimum length of service for retirement (MLOS) is 20 years. The analysis examined the impact of adjusting the minimum retirement upward to 30 years in 1 year increments under two alternative attrition assumptions. Under alternative A it was assumed that if a paygrade's average length of service was less than the trial MLOS value, then the population of that paygrade was retained until the minimum retirement point with the condition that the population was decremented by 2 percent for each year of extension.[5] For example a First Class Petty Officer or Staff Sergeant, paygrade E6, normally retires at 21.0 years of service. If the trial MLOS value was 23.0 years of service, then under alternative A the population of retiring E6's would be reduced by $(.98)^2$ for the 2 years to a level of 96.04 percent of its previous retirement population. Under alternative B no reduction was made in the population of retiring E6's. These are arbitrary decrements which yielded mixed results as displayed in Table 3. The two alternatives are believed to bracket the actual leaving rates that would be encountered under changes in minimum length of service to retirement. The baseline configuration with a total annual retirement cost of \$1,210M is associated with the minimum length of service for retirement value of 20 years.

TABLE 3

MINIMUM LENGTH OF SERVICE EFFECTS

Length of Service Required to Retire	Total Annual Retirement Cost	
YRS	A	B
20	$1,210M	$1,210
21	1,210	1,210
22	1,250	1,260
23	1,250	1,270
24	1,250	1,280
25	1,250	1,300
26	1,290	1,360
27	1,270	1,370
28	1,260	1,380
29	1,230	1,380
30	1,210	1,380

A: attrition rate of 0.02 per year after 20 years

B: no attrition after 20 years

Under alternative A the data can be addressed as 3 clusters. There is no effect on total annual retirement cost for an addition of 1 year (raising the minimum length of service to 21 years). Between the years 22 and 25, there appears to be no discernible pattern. Years 26 through 30 show a decreasing trend. Under alternative B there is a monotonic increase in total annual retirement cost from $1,210 M under 20 years minimum service for retirement to $1,380 M for 30 years minimum service for retirement. The intriguing result under alternative B is that increasing the minimum length of service to be eligible for retirement <u>increases</u> total annual retirement costs. This is due to the increase in percent of base pay that occurs with each year's addition past 20 years of service.

No functional relationship meeting the R^2 criteria of .90 was found for alternative A. For alternative B, however, an R^2 of .940 was calculated for the linear function TARC = 818.9 + 19.6 MLOS.

Length of Service at Retirement

For the sensitivity analysis of the length of service at retirement (LOSD) variable, the variable value represents the value of the adjustment to the Department of Defense Actuary's estimates. The average length of service at retirement for each pay grade was examined over a range of 3 years less than the current average to 3 years more than the current average, in .6 year increments.[6]

The baseline configuration with a TRC of $1,120M is associated with an LOSD value of 0 (meaning no adjustment to the average length of service at retirement). The analysis of the total retirement cost changes resulting from the variable indicate that the larger retirement annuity awarded for an increased length of service is largely offset by the decreased life expectancy of the later retirement. An increased liability of only $60M was incurred when the average length of service was increased by 2 years, and a further increase to 3 years resulted in virtually no increase in total retirement cost beyond that of the 2 year extension. An R^2 of .952 was calculated from the linear function TRC = $1,219 + ($19.7 x LOSD).

Life Expectancy at Retirement

Sensitivity analysis of the life expectancy at retirement (LEXPD) was an incremental analysis done in a manner similar to the length of service at retirement. However, only increases to life expectancy were analyzed, since in the United States, life expectancies have shown a strong tendency to increase [2]. Therefore, life expectancy at retirement adjustments ranging from increases of 0 to 5 years in .5 year increments were explored.

The relationship of total annual retirement costs to the increasing life expectancy at retirement appears to be linear. Minor increases in annual retirement cost was anticipated since the annuity lengths supported by the

life expectancy average approximately 32 years. At that distance from initial funding, adjustments to the annuity lengths do not require equal increases in costs recognized. An R^2 of .994 was associated with the linear function TRC = $1,214 + ($5.54 x LEXPD).

Entrant Retirement Probability

For the entrant retirement probability the increments are changes in the baseline probability. Entrant retirement probability (ERPD) adjustments of .03 less than the current average probability to .03 more than the current average probability were examined in .01 increments. The baseline configuration with a TRC of $1,120M is the ERPD of 0 (meaning no adjustment to entrant retirement probability). When the total retirement cost was regressed on entrant retirement probability the linear function TRC = $1,212 + ($71.7 x ERPD) yielded an R^2 of .999.

Each of the equations were developed from eleven data parts. Usually, such small samples as used in the regression analysis of the sensitivity results are of limited value. However, coefficients of determination as large as those in Table 2 imply that the regression line is a good approximation of the analytical relationship over the relevant range between TARC and the associated variable. This method has fortunately generated apparently reliable approximations to relationships that would have been very difficult to solve analytically.

RELATIVE IMPACT OF INDIVIDUAL VARIABLES

The equations derived in the previous section can be used to estimate the relative impact of changes in the variables on future military retirement costs. The variables are listed in Table 4 in the order of most sensitive to

least sensitive as measured by the impact on total annual retirement costs of a 10 percent change in the baseline input value of each variable. Listings under the "Change in Total Annual Retirement Cost" heading show the difference in millions of dollars of total annual retirement cost resulting from each 10 percent change. The associated percentage variation in baseline total annual retirement cost caused by the incremental change are listed in the next column. The heading "Controllability" refers to the previously discussed controllability (C) or noncontrollability (N) of the variable from the perspective of the Department of Defense.

Two important caveats are necessary for properly interpreting this table. First, the relative ranking of elasticity values is not necessarily the ranking of relative importance. A 10 percent change in length of service at retirement yields a 3.8 percent change in total retirement costs while a 10 percent change in percent of base pay at retirement yields a 10 percent change in total retirement. However, if a 10 percent change in length of service at retirement is three times more likely to occur than a 10 percent change in percent of base pay at retirement, then it may be argued that length of service at retirement is more important than percent of base pay at retirement. If large changes began to occur in uncontrollable factors such that total force structure was being adversely affected, in all likelihood compensating changes in controllable factors would be undertaken in order to maintain the desired rank, paygrade and total size goals of the Department of Defense. For example, with respect to the annual discount rate, if changes in annual discount rates were beginning to have substantial effects on force size or force structure, then compensatory changes in current bonuses and salaries would probably be undertaken to maintain force size or force structure goals.

TABLE 4

INDIVIDUAL ENTRY AGE NORMAL SENSITIVITY RANKING*

Variable	DOD Controllability	Change in Total Annual Retirement Cost[a]	Percentage Change in Total Annual Retirement Cost[b]
Annual Discount Rate	N	-$297.4M	-24.5%
Percent of Base Pay at Retirement	N	121.1	10.0
Entrant Retirement Probability	C	114.9	9.5
Estimated Rate of Salary Increase	N	95.9	7.9
Length of Service at Retirement	C	46.5	3.8
Length of Service Required to Retire	N	43.4	3.6
Life Expectancy at Retirement	N	12.2	1.0
Maximum Allowed Percent of Base Pay	C	2.0	.1

* from a 10% change in the baseline input value for each variable.
[a] The change in Total Annual Retirement Cost from the baseline value of $1,210M.
[b] The change in Total Annual Retirement Cost divided by the baseline Total Annual Retirement Cost value.

Of the variables controllable at the level of the DOD, adjustments to the entrant retirement probability seem to offer the most promise in managing retirement costs. It is interesting to note that an increase in length of service at retirement of two years (10%) has a surprisingly low corresponding retirement cost increase. Efforts to increase retention after retirement eligibility is reached would increase the overall experience level of the

service and (assuming experienced personnel perform better than inexperienced) may produce a superior performance to cost ratio. Adjustments to length of service required to retire has approximately the same impact on total annual retirement cost as the length of service at retirement variable.

The second caveat is that each of the variable values which comprise the baseline configuration differ in their inherent accuracy. The discount rate (DIS%) and rate of salary increase (SAL%), being functions of future economic performance, are both "soft" numbers in which excessive confidence should not be placed. At the opposite end of the reliability spectrum are values for percent of base pay at retirement (PAY%), maximum allowed percentage of base pay (MAX%), and minimum length of service to retire (MLOS). Each of these values is fixed in law and therefore will probably remain constant for the long term. Confidence in variable values for retention until retirement (ERPD), average length of service at retirement (LOSD), and life expectancy at retirement (LEXPD) lies between the two ends of the spectrum discussed above. Although only estimates, they have been subjected to actuarial review.

JOINT EFFECTS

An additional use of the regression approach for simply and quickly predicting effects on system performance of changes in parametric values is to estimate the complex joint effects interrelationships imbedded in the full simulation model. Given the equations for the simple effects (Table 2), the joint effects can be easily estimated. For instance, referring to Table 4, Annual Discount Rate and Percent of Base Pay at Retirement have the highest simple effects on changes in total retirement cost. If estimation of the

joint effects of the two variables is desired, it is simply a matter of combining the effects plus an interaction term, that is:

TRC = f(simple effect of DIS%, simple effect of PAY%, interaction of DIS% and PAY%)

In this particular case the annual Discount Rate (DIS%) was estimated by a logrithmic function and the effect of the changes in Percent of Base Pay at Retirement (PAY%) were estimated directly from the PAY% data. Given these relationships the functional form of the regression equation would be:

$$TRC = B_0 + B_1(\text{Log DIS\%}) + B_2(\text{PAY\%}) + B_3(\text{Log DIS\%})(\text{PAY\%})$$

The estimates of the B's in the regression equation were developed in the same manner as those for the simple effects. That is, the full simulation was run changing the variables of interest, in this case DIS% and PAY%. In order to insure that the effects of the interaction were captured the best case-worst case senarios were used. The simulation model was run with the two highest values for DIS% and PAY%, the two lowest values for DIS% and PAY% and the high-low combinations or as follows:

	DISC%	PAY%
High/High	.15	.03
Low/Low	.05	.02
High/Low	.15	.02
Low/High	.05	.03

Additionally, random combinations of the two variables within the appropriate ranges were generated using a normal distribution centered on the most likely value. In all twenty observations were made for joint effects of

the two variables. These data then were regressed to generate the following equation yielding an R^2 of .928:

$$TRC = -0.2562 + 0.1916(\text{Log DIS\%})-182.8025(\text{PAY\%})-239.9817(\text{Log DIS\%xPAY\%})$$

This same method could be used to identify the interaction effects for other combinations of interest. For the particular example used in this paper, interaction effects were not of concern for estimating purposes.

CONCLUSION

The generic procedures discussed above present guidelines for using a regression procedure to estimate equations that simply and quickly predict the impact of changes in parameter values of complex relationships embedded in a mathematical simulation model. The steps involved include establishment of baseline values for all variables, selection of individual variables for analysis, identification of the functional form of system performance and individual variable relationship, selection of range and step values, iteratively running the simulation model for sample individual variable values to generate data for regression analysis, and estimation of the parametric equations.

The estimating relationship derived from a sensitivity analysis of a simulation model can be used by policy makers to estimate the effects on total system performance of a change in one of the primitive variables even though the policy maker does not have hands-on access to the system computer simulation model. In the retirement cost example presented above the

equations serve as a useful method for quickly estimating the impact of such changes when the recalculation of the entry age normal model is not feasible, such as during budget negotiations.

The same methodology which produced the estimating equations for the retirement cost simulation model can be applied to economic, managerial or legal input variable assumptions in simulation models to provide decision makers with easy to use system performance estimators.

NOTES

1. The military retirement system is a defined benefit plan with voluntary non-disability retirement authorized after 20 years of active military service. The retiree receives 2.5 percent of active duty base pay for each year of service up to 30 years. Depending upon date of original enlistment, retired pay is either calculated using the final basic pay or the last 36 months of basic pay. For Fiscal Year 1981 the outlays for current retirees were $12.5 billion. (Aeila, 1980; U.S. Department of Defense, 1976; U.S. Department of Defense 1976).

2. A full description of the variables and model can be obtained upon request to the authors.

3. The 9 percent value also compares more favorably with the interest rates promulgated by the Department of the Treasury pursuant to Public Law 92-41. This is the interest rate used by government estimators when performing cost calculations which require a government cost of money. This figure is actually a complex average of both government and low risk private securities with 5 year maturities. Since it is a medium term number, the 9 percent long term number was considered a better estimator.

4. The manpower figures used in the calculations include only regular Navy (USN), enlisted and officer personnel for the years 1953 through 1982.

The calculated TRC includes neither disability nor survivor benefits, which are not retirement costs in the strict sense, but the result of military self-insurance and therefore not included in the individual entry age normal computations.

5. The selection of 2% was arbitrary. A smaller percent loss would result in greater annuity costs. Given the small loss ratio that is experienced in years of service 6 to 20 when minimum length of service to retirement is 20 years, 2% is a reasonable upper bound for losses prior to a new minimum length of service required for retirement.

6. An important feature of the length of service at retirement computation is that if the input of a negative LOSD would reduce a pay grade's average length of service to less than the minimum required for retirement (currently 20 years), the input is disallowed, and the length of service is reduced only to the minimum required for retirement. A corresponding approach has been taken to the problem of age at retirement. Any retirement at less than age 37 is not allowed, since the minimum acceptable age for entry into the armed forces is 17 years and 20 years of service are required to retire (17 + 20 = 37).

REFERENCES

[1] AEILA (1980) "Military Retirement, The Administration's Plan and Related Proposals." Washington, D.C.: American Enterprise Institute for Public Research.

[2] American Council of Life Insurance (1982), 1982 Life Insurance Fact Book. Washington, D. C.

[3] CBO (1978) "The Military Retirement System: Options for Change." Washington, D.C.: U.S. Government Printing Office.

[4] Dreher, W.A. September (1967) "Alternatives Available Under APB Opinion No. 8: An Actuary's View," The Journal of Accountancy: 37-51.

[5] Friedman, L. and H. Friedman (1984) "Statistical Considerations in Computer Simulation: The State of the Art," Journal of Statistical Computation and Simulation, Vol. 19,: 237-263.

[6] Kleijnen, Jack P.C. (1979) "Regression Metamodels for Generalizing Simulation Results," IEEE Transactions on Systems, Man, and Cybernetics, Vol. SMC-9, No.2.

[7] Kleijnen, Jack P.C. (1975) "A Comment on Blanning's "Metamodel for Sensitivity Analysis: The Regression Metamodel in Simulation," Interfaces, Vol. 5, No. 3.

[8] Smith, D.F., September (1983) "A Sensitivity Analysis of Entry Age Normal Military Retirement Costs, Unpublished Master's Thesis." Naval Postgraduate School.

[9] Tukey, J.W. (1977) Exploratory Data Analysis, Adison-Wesley, Reading, MA.

[10] U.S. Department of Defense (1976) "Third Quadrennial Review of Military Compensation: Modernizing Military Pay," Washington, D.C.: U.S. Government Printing Office.

[11] U.S. Department of Defense (1979) Military Retired Pay Manual, DoD Manual 1340.12-M, Washington, D.C.

[12] U.S. Department of Defens (1983) "Valuation of the Military Retirement System," FY 1982, Office of the Actuary, Defense Manpower Data Center Alexandria, VA.

[13] U.S. Department of the Treasur (Summer 1983) Treasury Bulletin, Washington, D.C.: Office of the Secretary.

[14] Waterman, Steven (June 1983) "Allocation Methods for Use in the Accrual of Manpower Costs," Unpublished Master's Thesis, Naval Postgraduate School.

APPENDIX A

TABLE A-1

GENERAL INDIVIDUAL ENTRY AGE NORMAL VARIABLES

Description	Variable
Annual Retired Annuity	A
Actuarial Normal Cost	AC
Annual Discount Rate	DIS
Current Year Gains/Losses	F
Current Year Applied Gains/Losses	Fa
Deferred Gains/Losses	Fd
Life Expectancy at Retirement	LEXP
Number of Contribution Years	n
Annual Normal Cost	NC
Present Value of Retirement Benefits	P
Current Year's Retirement Cost	RC
Remaining Working-Life of Employee	RWL

TABLE A-2

MILITARY INDIVIDUAL ENTRY AGE NORMAL VARIABLES

Description	Variable
Final Monthly Base Pay	BP
Current Base Pay at Retirement Grade	PBc
Paygrade of Retiree	G
Number of Entrants for a Given Year	I
Length of Service at Retirement	LOS
Minimum Length of Service Required to Retire	MLOS
Expected Number of Retirees	N
Probability of an Entrant Retiring at a Given Paygrade, G	Pr(G)
Probability of an Entrant Reaching Retirement,	ERP
Retirement Percentage Rate	RR
Percent of Base Pay at Retirement	PAY%
Maximum Allowed Percent of Base Pay	MAX%
Estimated Rate of Salary Increase	SAL%
Current Total Retirement Cost	TRC

DISTRIBUTION LIST

	No. of Copies
Kenneth J. Euske Code 54Ee Naval Postgraduate School Monterey, CA 93943	5
George W. Thomas Code 54Te Naval Postgraduate School Monterey CA 93943	25
Director of Research Code 012 Naval Postgraduate School Monterey, CA 93943	1
Dudley Knox Library Code 0212 Naval Postgraduate School Monterey, CA 93943	2
Defense Technical Information Center Cameron Station Alexandria, VA 22314	2
LCDR D.F. Smith USS Belknap (CG 26) FPO New York 09565-1149	2

DTIC
FILMED
4-86
END